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488 (Algebra). Proposed by NORMAN ANNING, Chilliwack, B. C.

Show that

$$\begin{vmatrix} a_1 & 1 & a_2 \\ 1 & 1/a_4 & 1 \\ a_2 & 1 & a_3 \end{vmatrix} = 0,$$

where

$$a_k = \frac{\sin (k\theta + \alpha)}{\sin k\theta},$$

and θ and α have any values that do not make a denominator zero.

Solution by A. M. Harding, University of Arkansas.

It is easily shown that the following equations hold for all values of θ:

$$\cot \theta - 2 \cot 2\theta + \cot 3\theta = \frac{2 \cos 2\theta}{\sin 4\theta + \sin 2\theta},$$

$$\cot \theta \cot 3\theta - \cot^2 2\theta = \frac{2 \cos 2\theta \cot 4\theta}{\sin 4\theta + \sin 2\theta};$$

that is

$$\cot \theta \cot 3\theta - \cot^2 2\theta = \cot 4\theta(\cot \theta - 2 \cot 2\theta + \cot 3\theta).$$

Now a_k may be written in the form

$$a_k = \cos \alpha + \sin \alpha \cot k\theta.$$

Thence we have

$$a_1 - 2a_2 + a_3 = \sin \alpha(\cot \theta - 2 \cot 2\theta + \cot 3\theta),$$

and

$$\begin{aligned} a_1a_3 - a_2^2 &= \sin \alpha \cos \alpha(\cot \theta - 2 \cot 2\theta + \cot 3\theta) + \sin^2 \alpha(\cot \theta \cot 3\theta - \cot^2 2\theta) \\ &= \sin \alpha(\cot \theta - 2 \cot 2\theta + \cot 3\theta)(\cos \alpha + \sin \alpha \cot 4\theta) \\ &= (a_1 - 2a_2 + a_3)a_4. \end{aligned}$$

Hence

$$\frac{a_1a_3 - a_2^2}{a_4} + 2a_2 - a_1 - a_3 = 0;$$

that is,

$$\begin{vmatrix} a_1 & 1 & a_2 \\ 1 & 1/a_4 & 1 \\ a_2 & 1 & a_3 \end{vmatrix} = 0.$$

Also solved by Paul Capron, Oscar Adams, and Edward H. Worthington.

489 (Algebra). Proposed by S. A. COREY, Albia, Iowa.

Prove or disprove the following:

$$\begin{vmatrix} -x & -ay & -bu & abv \\ y & x & -bv & -bu \\ u & av & x & ay \\ -v & -u & y & -x \end{vmatrix}^2 + a\begin{vmatrix} x & -x & -bu & abv \\ y & y & -bv & -bu \\ u & u & x & ay \\ v & -v & y & -x \end{vmatrix}^2$$

$$+ b\begin{vmatrix} x & -ay & -x & abv \\ y & x & y & -bu \\ u & av & u & ay \\ v & -u & -v & -x \end{vmatrix}^2 + ab\begin{vmatrix} x & -ay & -bu & -x \\ y & x & -bv & y \\ u & av & x & u \\ v & -u & y & -v \end{vmatrix}^2 = \begin{vmatrix} x & -ay & -bu & abv \\ y & x & -bv & -bu \\ u & av & x & ay \\ v & -u & y & -x \end{vmatrix}^2.$$